Reflection at a Conducting Surface

Boundary conditions in presence of free charges and currents

(i) $\epsilon_1 E_1^{\perp} - \epsilon_2 E_2^{\perp} = \sigma_f$ (iii) $E_1^{\parallel} - E_2^{\parallel} = 0$

(ii) $B_1^{\perp} - B_2^{\perp} = 0$ (iv) $\frac{1}{\mu_1} B_1^{\parallel} - \frac{1}{\mu_2} B_2^{\parallel} = K_f \times \hat{n}$ --- (22)

$\sigma_f \rightarrow$ free surface charge

$K_f \rightarrow$ free surface current

$\hat{n} \rightarrow$ unit vector perpendicular to the surface pointing from medium (2) to (1)

For ohmic conductors ($J_f = \sigma E$) $\rightarrow$ no free surface current, since it would require an infinite electric field at the boundary.

xy plane forms the boundary between a non-conducting linear medium (1) and a conductor (2).

A monochromatic plane wave travelling in the z-direction and polarized in the x-direction approaches from the left

$$\tilde{E}_I(z,t) = \tilde{E}_{0I} e^{i(k_1 z - \omega t)} \hat{x}, \quad \tilde{B}_I(z,t) = \frac{1}{v_1} \tilde{E}_{0I} e^{i(k_1 z - \omega t)} \hat{y} \quad \text{--- (23)}$$

Scanned by CamScanner

Reflected wave

$$\tilde{E}_R(z,t) = \tilde{E}_{0R} e^{i(-k_1 z - \omega t)} \hat{x}, \quad \tilde{B}_R(z,t) = -\frac{1}{v_1} \tilde{E}_{0R} e^{i(-k_1 z - \omega t)} \hat{y}$$

Propagating back in medium (1) —(24)

and Transmitted wave

$$\tilde{E}_T(z,t) = \tilde{E}_{0T} e^{i(\tilde{k}_2 z - \omega t)} \hat{x}, \quad \tilde{B}_T(z,t) = \frac{\tilde{k}_2}{\omega} \tilde{E}_{0T} e^{i(\tilde{k}_2 z - \omega t)} \hat{y}$$

→ attenuated as it penetrates into the conductor —(25)

at $z=0$, the combined wave in medium (1) must join the wave in medium (2), according to the boundary conditions (22)

(i) gives $\sigma_f = 0$ since $E^\perp = 0$ on both sides

Since $B^\perp = 0$, (ii) is satisfied

(iii) gives

$$\tilde{E}_{0I} + \tilde{E}_{0R} = \tilde{E}_{0T} \quad \text{—— (26)}$$

and (iv) (with $K_f = 0$) gives

$$\frac{1}{\mu_1 v_1}(\tilde{E}_{0I} - \tilde{E}_{0R}) - \frac{\tilde{k}_2}{\mu_2 \omega} \tilde{E}_{0T} = 0 \quad \text{—— (27)}$$

or

$$\tilde{E}_{0I} - \tilde{E}_{0R} = \tilde{\beta} \tilde{E}_{0T} \quad \text{—— (28)}$$

where

$$\tilde{\beta} \equiv \frac{\mu_1 v_1}{\mu_2 \omega} \tilde{k}_2 \quad \text{—— (29)}$$

$$\Rightarrow \tilde{E}_{0R} = \left(\frac{1-\tilde{\beta}}{1+\tilde{\beta}}\right) \tilde{E}_{0I}, \quad \tilde{E}_{0T} = \left(\frac{2}{1+\tilde{\beta}}\right) \tilde{E}_{0I} \quad \text{—— (30)}$$

Scanned by CamScanner

$\beta \rightarrow$ now a complex number

for a perfect conductor ($\sigma = \infty$), $k_2 = \infty$,

so $\tilde{\beta}$ is infinite, and

$$\tilde{E}_{0R} = -\tilde{E}_{0I}, \quad \tilde{E}_{0T} = 0 \quad \text{---} (31)$$

Wave is totally reflected, with a 180° phase shift.

$\Rightarrow$ Excellent conductors make good mirrors.

Thin coating of silver onto the back of a pane of glass, skin depth of silver at optical frequencies is on the order of $100\,A^\circ$, not very thick layer is required.

Ex: Find the phase velocity and the magnitude of the attenuation constant of plane waves at a frequency 10 GHz in polyethelene, given

$\mu = \mu_0$, $\varepsilon_r = 2.3$ and $\sigma = 2.56 \times 10^{-4}$ mho/m

Sol^n phase velocity $v = \dfrac{1}{\sqrt{\mu\varepsilon}} = \dfrac{\sqrt{\varepsilon_0/\varepsilon}}{\sqrt{\mu_0\varepsilon_0}} = c\sqrt{\varepsilon_0/\varepsilon}$

$$= c\sqrt{1/\varepsilon_r} = 3\times10^8 \times \sqrt{\frac{1}{2.3}}$$

$$= 1.97\times10^8\ ms^{-1}$$

Since $\dfrac{\sigma}{\omega\varepsilon} = \left(\dfrac{2.56\times10^{-4}}{2\pi\times10^{10}\times2.3\times8.85\times10^{-12}}\right) = 2\times10^{-4}$

$$\frac{\sigma}{\omega\varepsilon} << 1$$

$$K = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\left\{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2\right\}^{\frac{1}{2}}-1\right]^{\frac{1}{2}}$$

$$\Rightarrow K = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\frac{\sigma^2}{2\varepsilon^2\omega^2}\right]^{\frac{1}{2}}$$

$$\Rightarrow K = \frac{1}{2}\sigma\sqrt{\frac{\mu}{\varepsilon}} = \frac{1}{2}\frac{\sigma}{\omega\varepsilon}\cdot\omega\sqrt{\mu\varepsilon}$$

$$= \frac{1}{2}\frac{\sigma}{\omega\varepsilon}\frac{\omega}{v}$$

$$= \frac{1}{2}\times 2\times10^{-4}\times 2\pi\times10^{10}\times\frac{1}{1.97\times10^{8}}$$

$$= \underline{3.18\times10^{-2}}$$

<u>Prob.</u> Calculate the reflection coefficient for light at an air to silver interface ($\mu_1 = \mu_2 = \mu_0$, $\varepsilon_1 = \varepsilon_0$, $\sigma = 6\times10^7\,(\Omega\text{-m})^{-1}$) at optical frequencies ($\omega = 4\times10^{15}$ /s).

Hint- reflection coefficient

$$R = \frac{|E_{0R}|^2}{|E_{0I}|^2}$$

Scanned by CamScanner